Conservation Laws for Counter-Propagating Optical Waves in Atmospheric Turbulence with Application to Directed Energy and Laser Communications

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Abstract: The relation between two waves propagating in opposite directions – the so-called bidirectional or counter-propagation- through a common path in turbulent atmosphere has been the subject of numerous theoretical and experimental studies. In most commonly considered double-pass and target-in-the-loop scenarios, a transmitted optical wave propagates through atmosphere toward a remote object (target) and then propagates back after being scattered off the object's surface. The double-pass propagation results in several interesting effects including enhanced intensity and phase fluctuations of the backscattered (target-return) wave and enhanced correlation of power signals received at both ends of bidirectional point-to-point optical links. In this paper we discussed the integral relationships between the counter-propagating wave complex amplitudes known as overlapping integrals or interference metrics, which values are preserved along the propagation path. We show that the conservation property of these integral quantities of the counter-propagating waves can be utilized for atmospheric turbulence effects mitigation in directed energy, free-space laser communication and active imaging applications.

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1. Introduction: Conservation integrals for counter-propagating waves

Consider the counter-propagating waves with complex amplitudes $A(\mathbf{r},z,t)$ and $\psi(\mathbf{r},z,t)$, where $0 \le z \le L$ is a coordinate colinear to propagation direction (optical axis), $\mathbf{r}=\{x,y\}$ is coordinate vector in the plane orthogonal to the optical axis, L is propagation distance and t is time. For simplicity, we assume that both optical waves are monochromatic with identical wavelength λ. In the quasi-optical approximation, propagation of these waves can be described by the following system of parabolic equations [1]:

$$2ik\frac{\partial A(\mathbf{r},z,t)}{\partial z} = \nabla_\perp^2 A(\mathbf{r},z,t) + 2k^2 n_1(\mathbf{r},z,t)A(\mathbf{r},z,t) \tag{1}$$

$$-2ik\frac{\partial \psi(\mathbf{r},z,t)}{\partial z} = \nabla_\perp^2 \psi(\mathbf{r},z,t) + 2k^2 n_1(\mathbf{r},z,t)\psi(\mathbf{r},z,t) \tag{2}$$

where $\nabla_\perp^2 = \partial^2/\partial x + \partial^2/\partial y$ is the Laplacian operator over the transversal coordinates, $n_1(\mathbf{r},z,t)$ is a function describing the refractive index fluctuations, $k = k_0 n_0$, $k_0 = 2\pi/\lambda$ is the wavenumber, and n_0 is refractive index undisturbed value. We assumed here that refractive index inhomogeneities can be considered as frozen during optical wave propagation over distance L, that is $L < c\,\tau_{at}$, where c and τ_{at} are the speed of light and atmospheric characteristic time respectively.

From the system of Eqns (1), (2) one can obtain the following conservation law, that links the counter-propagating wave complex amplitudes and known as the interference metric or overlapping integral [2-5]

$$J_{\mathrm{int}}(t) = \int A(\mathbf{r},z,t)\psi(\mathbf{r},z,t)d^2\mathbf{r} = f(t), \tag{3}$$

where $f(t)$ is an independent on variable z function. Thus, at a fixed time t the integral (3) has the same value at both ends of the propagation path:

$$J_{\mathrm{int}}(t) = \int A(\mathbf{r},z=0,t)\psi(\mathbf{r},z=0,t)d^2\mathbf{r} = \int A(\mathbf{r},z=L,t)\psi(\mathbf{r},z=L,t)d^2\mathbf{r}\,. \tag{4}$$

Note that expression (4) is valid for arbitrary counter-propagating waves.
Consider the counter-propagating waves in the target-in-the-loop laser beam propagation scenario as in Fig. 1 and calculate the overlapping integral $J_{\text{int}}(t)$ at the transceiver (z=0), and target (z=L) planes. In this case the target-return wave $\psi(\mathbf{r}, z = L)$ in Eqn (4) is defined by the laser beam scattering off the target surface. Represent the scattering condition in the form $\psi(\mathbf{r}, z = L) = T(\mathbf{r})A(\mathbf{r}, z = L)$, where $T(\mathbf{r})$ is target complex scattering coefficient.
By substituting the scattering conditions into Eqn (4), we obtain

$$J_{\text{int}}(t) = \int A(\mathbf{r}, z = 0, t)\psi(\mathbf{r}, z = 0, t)d^2\mathbf{r} = \int T(\mathbf{r})A^2(\mathbf{r}, z = L, t)d^2\mathbf{r}\ . \qquad (5)$$

Expression (5) couples the optical wave's characteristics at the transceiver and target planes, and thus offers opportunity for quality evaluation of the target-plane laser beam power spatial distribution (quality of the target hit-spot brightness) using measurements of the interference metric at the transceiver plane. In this presentation we provide several examples illustrating how optimization of the interference metric can result in target hit-spot brightness increase and hence can be used for the outgoing beam control and adaptive optics (AO) atmospheric effects mitigation in laser beam projection applications.

2. Interference metric sensing with a single-mode fiber based optical receiver

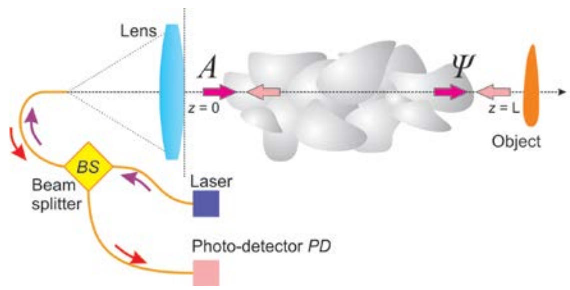

Fig. 1: Notional schematic of the target-in-the-loop bidirectional wave propagation link based on single-mode fiber-collimator transceiver composed of a collimating lense with fiber tip located in its focus and optical trains based on single-mode fibers for both transmitted and received waves.

The interference metric $J_{\text{int}}(t)$ can be directly measured using interference of the outgoing and scattered waves that are registered at the transceiver plane z=0 [4]. This method of the interference metric measurements requires utilization of a laser source with long coherence length. Another, practically more convenient method of interference metric sensing is discussed in [6]. It is shown that for a laser beam projection system that utilizes transceiver telescope with a single-mode fiber input as shown in Fig. 1, the power signal P registered by the fiber-coupled photo-detector is proportional to $|J_{\text{int}}(t)|^2$, that is

$$P(t) = \kappa\,|J_{\text{int}}(t)|^2 = \kappa\left|\int T(\mathbf{r})A^2(\mathbf{r}, z = L, t)d^2\mathbf{r}\right|^2 . \qquad (6)$$

Note that for the case of laser beam projection onto a planar mirror surface $T(\mathbf{r})$=T_0=const, and hence $P(t) = \kappa\,|J_{\text{int}}|^2 = \kappa T_0 \left|\int A^2(\mathbf{r}, z = L, t)d^2\mathbf{r}\right|^2$. We show that for relatively short propagation distances $L \le 0.3L_{dif}$, where $L_{dif} = ka_0^2/2$, and a_0 is the transmitted beam radius, optimization of the interference metric module potentially results in a hit-spot that exceeds the diffraction-limited beam size by less than 5%.

3. Adaptive beam projection onto an extended target with rough surface

Consider AO wavefront control based on interference metric optimization for the case of an extended target with randomly rough surface in an optically inhomogeneous medium. The complex scattering coefficient $T(\mathbf{r})$ can then be represented in the form $T(\mathbf{r}) = V(\mathbf{r})\exp[ik_z^s\xi(\mathbf{r})]$, where $\xi(\mathbf{r})$ is a random target surface profile function and k_z^s is the scattering vector component [7]. For simplicity we assumed that variations in the incident wave's wavefront slopes at the target surface can be neglected. The function $V(\mathbf{r}) = \gamma(\mathbf{r})\exp[ikS(\mathbf{r})]$ corresponds to the scattering coefficient in the absence of roughness, where $S(\mathbf{r})$ and $\gamma(\mathbf{r})$ are functions describing target shape and surface reflection characteristics. Using the introduced expression for the complex scattering coefficient the interference metric can be represented in the form

$$J_{\text{int}}(t) = \int A(\mathbf{r}, z = 0, t)\psi(\mathbf{r}, z = 0, t)d^2\mathbf{r} = \int V(\mathbf{r})\exp[ik_z^s\xi(\mathbf{r})]A^2(\mathbf{r}, z = L, t)d^2\mathbf{r}\ , \qquad (7)$$

Because of the presence of the random function $\xi(\mathbf{r})$, the interference metric in Eqn (7) is a random variable. This suggests that some processing of the interference signal should be performed to eliminate dependence of $J_{\text{int}}(t)$ on the surface roughness. For rapidly spinning targets or fast steering of the outgoing beam on the target surface the simplest processing of this type can include averaging the interference metric over an ensemble of surface roughness realizations [4].

Consider interference metric (7) averaged over an ensemble of random surface profile function realizations:

$$< J_{\text{int}}(t) >= \int V(\mathbf{r}) < \exp[ik_z^s \xi(\mathbf{r})] > A^2(\mathbf{r}, z = L, t) d^2\mathbf{r} , \tag{8}$$

In Eqn (8) averaging is performed over time τ_{av} << τ_{at} characterizing rapidly changing surface roughness realizations. It can be shown [4,8] that for flat object with $V(\mathbf{r})=V_0$ =const and Gaussian roughness

$$< J_{\text{int}}(t) >= V_0 \exp[-(\sigma_s k_z^s)^2 / 2] \int A^2(\mathbf{r}, z = L, t) d^2\mathbf{r} , \tag{9}$$

where σ_s is the standard deviation of the surface roughness. The factor $V_0 \exp[-(\sigma_s k_z^s)^2 / 2]$ in Eqn (9) defines the surface roughness characteristic attenuation coefficient. With an increase in the surface roughness σ_s the attenuation coefficient decreases. The averaged interference metric in Eqn (9) depends solely on the optical field distribution inside the target hit spot and, with accuracy up to a constant, coincides with the corresponding interference metric value for the case of laser beam projection onto the planar mirror surface. Thus, for an extended flat target, maximization of the averaged interference metric module by shaping the outgoing wave phase results in an intensity distribution on the flat target surface that is identical to that for a flat mirror surface obtained with the corresponding interference metric optimization.

Consider the laser beam projection system based on the single-mode fiber laser transceiver as in Fig. 1. In this system the measured target-return power signal P is proportional to the interference metric squared modulus. In the case of an extended fast spinning target or laser beam scanning the averaged over an ensemble of surface roughness realizations power signal (K-metric) is given by

$$K(t) =< P >=<| J_{\text{int}}(t) |^2 >=<| \int V(\mathbf{r}) \exp[ik_z^s \xi(\mathbf{r})] A^2(\mathbf{r}, z = L, t) d^2\mathbf{r} |^2 > . \tag{10}$$

It can be shown [8] that for a statistically uniform and isotropic random roughness $\xi(\mathbf{r})$ the expression for K metric can be simplified to

$$K(t) =< P >=<| J_{\text{int}}(t) |^2 >= \kappa_K \int | V(\mathbf{r}) |^2 \, I^2(\mathbf{r}, z = L, t) d^2\mathbf{r} , \tag{11}$$

where $\kappa_K = (\pi / 2)(k\theta_s)^2$ is a characteristic attenuation coefficient, and $I(\mathbf{r}, z = L, t) =| A(\mathbf{r}, z = L, t) |^2$ is the intensity distribution on the target surface. Here $\theta_s = \sigma_s / l_s$ and l_s are correspondingly the characteristic angle for the surface roughness slopes and the surface roughness correlation length. For an extended flat surface with $V(\mathbf{r})=V_0$=const K metric in Eqn (11) is proportional to the well-known sharpness function J_2. Thus, maximization of the K-metric measured at the receiver aperture leads to maximization of the sharpness function at the target surface, and correspondingly results in an increase in the target hit-spot brightness.

Besides the directed energy (beam projection) applications the conservation properties of the interference metric can be used for mitigation of turbulence-induced signal fading in laser communication and speckle modulation in active imaging systems [6].

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